R-OSSE
Acoustic Waveguide

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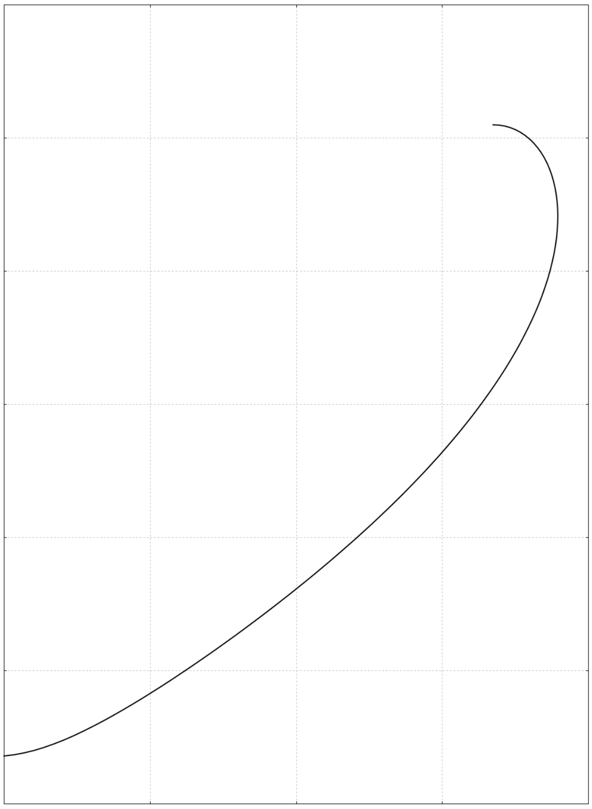


Introduction

In the previous work the OSSE ("OS-SE") waveguide formula was presented[1], extending the well-known Oblate Spheroidal (OS) waveguide by incorporating a smooth termination into a flat panel. While it proved the importance of the added gradual termination, due to its inherent half-space nature the usefulness was still somewhat limited - for a real-life use it is necessary to place such device into a finite baffle with an additional edge treatment which is not any more a part of its analytical description.

The now presented R-OSSE set of parametric equations goes a step further and defines a complete waveguide terminated into a free space by means of a convenient, self-containing analytical description. Such approach can be readily used e.g. in further optimization algorithms, CAD routines, etc.

The R-OSSE parametric description

In the following text we describe profile of an axisymmetric waveguide as a set of cartesian coordinates [x, y], where x is the axial distance from the throat and y is the distance of the profile point from the axis.

Because an OSSE profile has the form of a function y(x), it can't describe shapes that fold back as the profile curve progresses, i.e. it can't assign two different values of y for a single x. For this we need a parametric description in a form [x, y] = [x(t), y(t)], where x(t) and y(t) are functions of a new parameter (t). Typically these functions are constructed so that the parameter ranges from 0 to 1.

The functions used in the R-OSSE description are plotted in Fig 1.

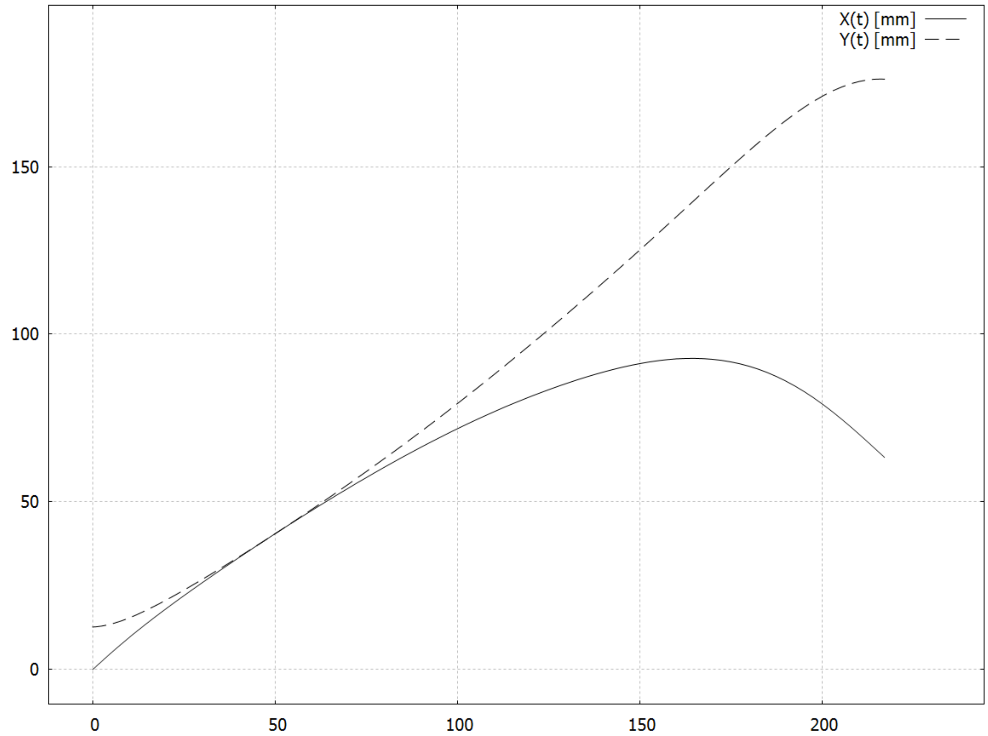


Fig. 1: R-OSSE x(t), y(t) components

1 http://www.at-horns.eu/release/OS-SE Waveguide.pdf

The functions in Fig. 1 are constructed by means of two conic sections each. The function x(t) is simply a difference of x1(t) and x2(t), a hyperbola and a parabola (Fig. 2). The function y(t) is a weighted average of y1(t) and y2(t), both being hyperbolas, starting as y2 and ending as y1 (Fig. 3). These functions were devised empirically, based on a good experience with profiles that, when decomposed into such functions, resembled similar shapes.

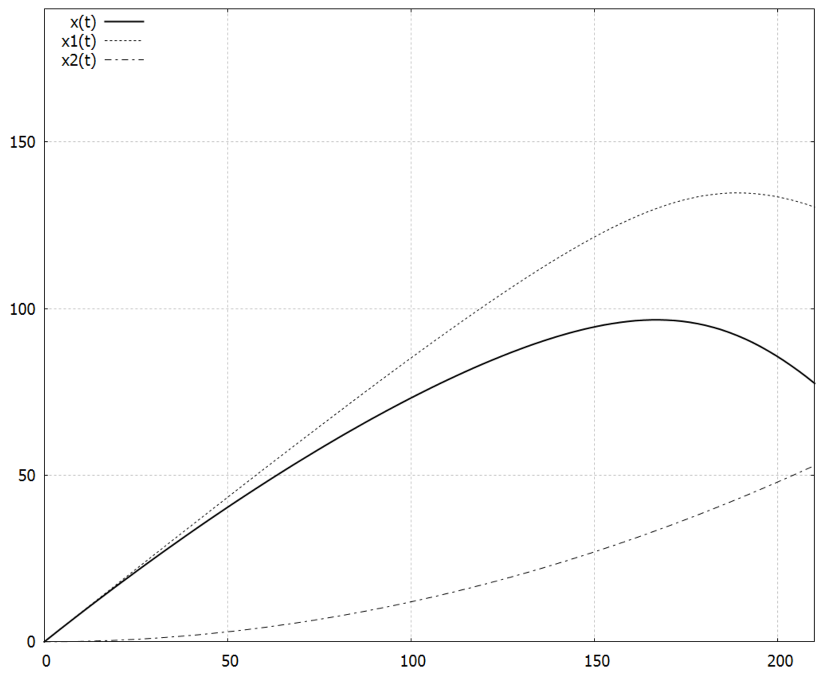


Fig. 2: x(t) components

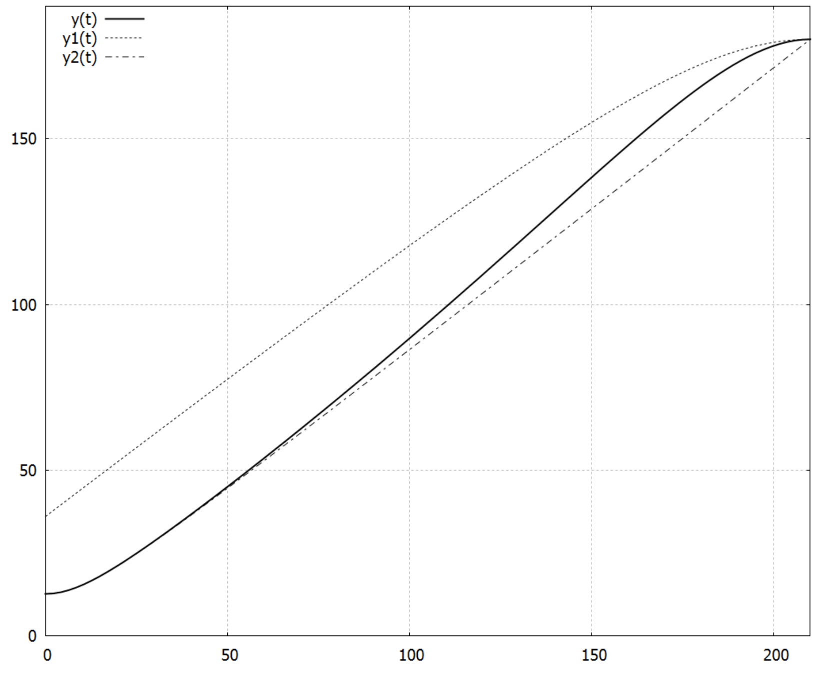


Fig. 3: y(t) components

R-OSSE design formulae

Design Parameter	Description	Unit
R	Waveguide outer radius	mm
a	Nominal coverage angle (*0.5)	deg
r_0	Throat radius	mm
a_0	Throat opening angle (*0.5)	deg
k	Throat expansion factor	--
r	Apex radius factor	--
m	Apex shift factor	--
b	Bending factor	--
q	Throat shape factor	--

Auxiliary constants:

$$c_1 = (kr_0)^2$$

$$c_2 = 2kr_0 \tan(a_0)$$

$$c_3 = \tan^2(a)$$

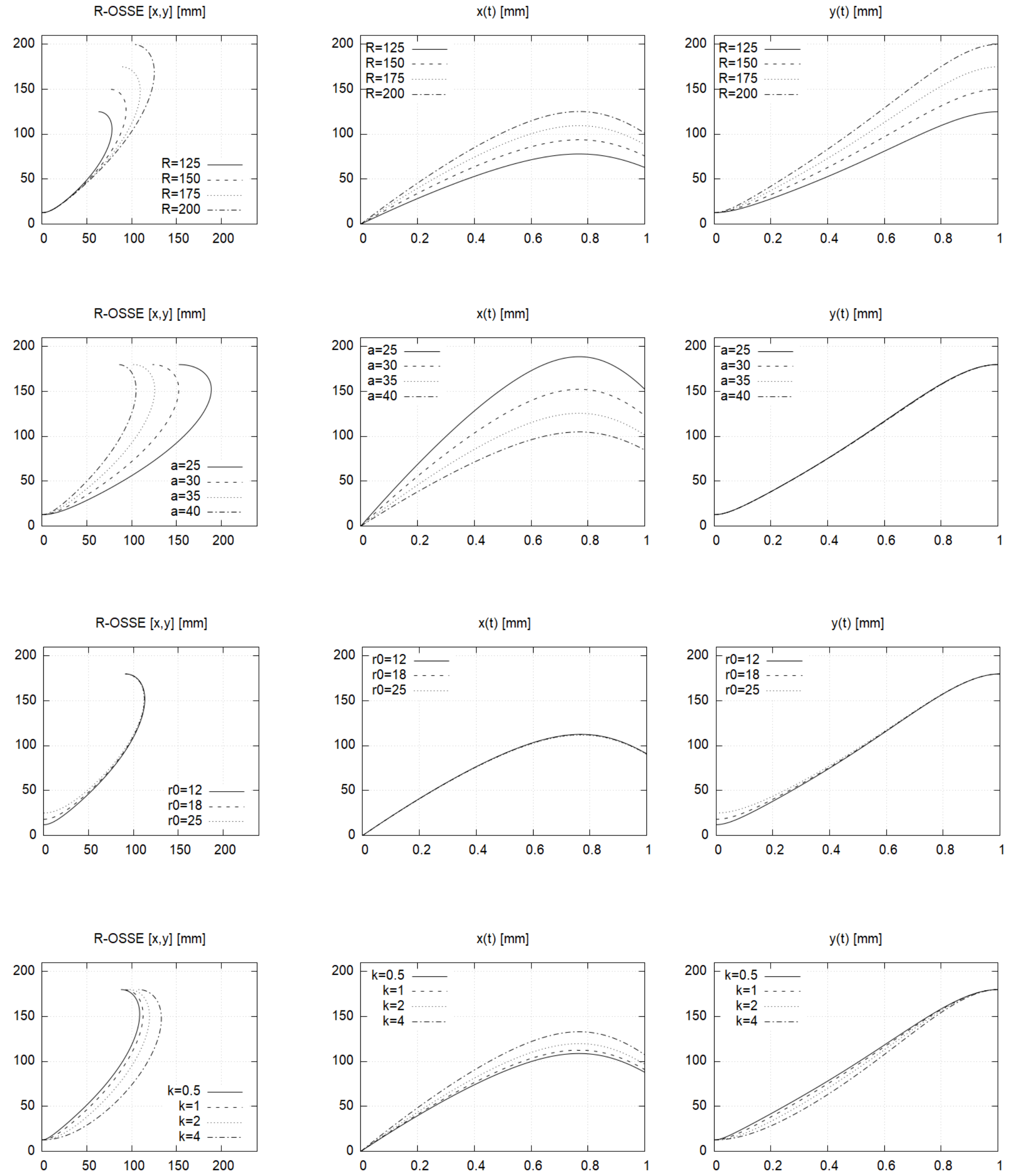
$$L = \frac{1}{2c_3}[\sqrt{c_2^2 - 4c_3(c_1 - (R + r_0(k-1))^2)} - c_2]$$

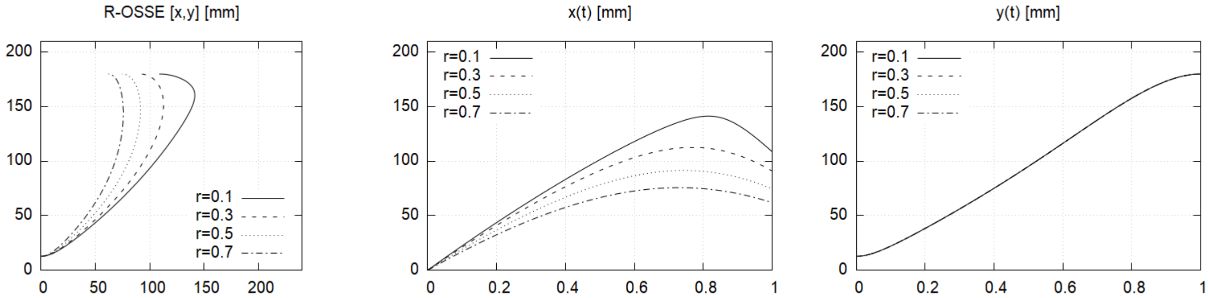
R-OSSE parametric formulae (for $0 \le t \le 1$):

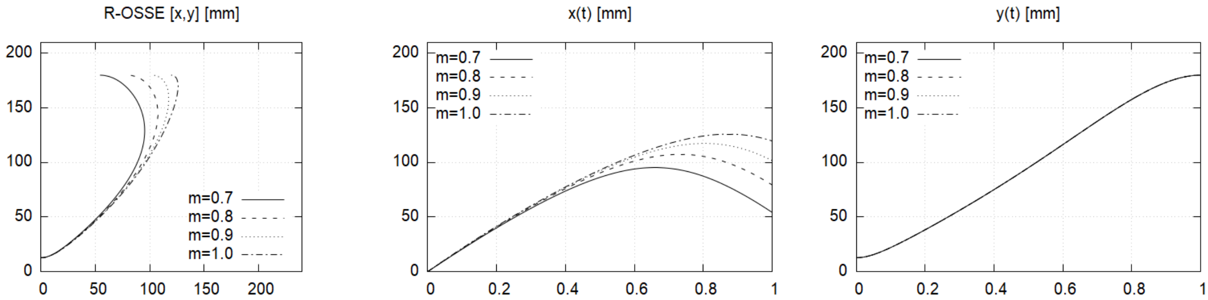
$$x(t) = L[\sqrt{r^2+m^2} - \sqrt{r^2+(t-m)^2}] + bL[\sqrt{r^2+(1-m)^2} - \sqrt{r^2+m^2}]t^2$$

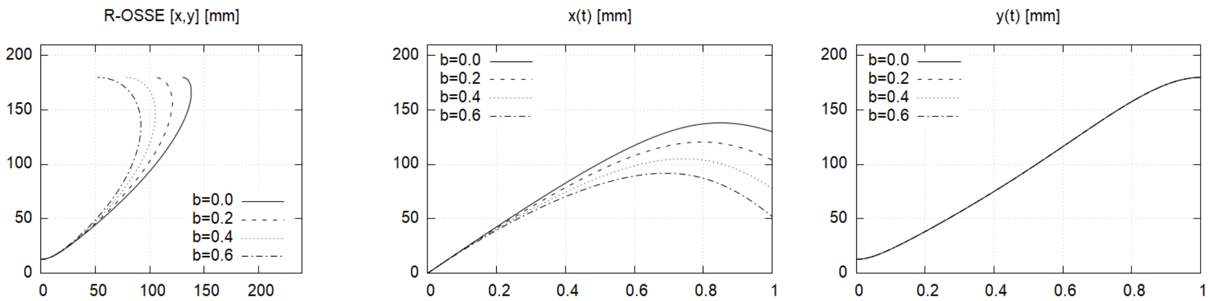
$$y(t) = (1-t^q)[\sqrt{c_1+c_2Lt+c_3L^2t^2} + r_0(1-k)] + t^q[R + L(1-\sqrt{1+c_3(t-1)^2})]$$

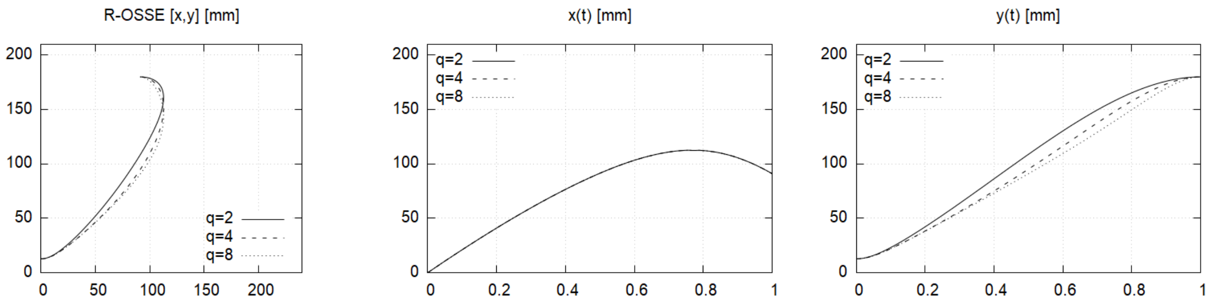
The following charts give an overview of the effect of each individual design parameter on the resulting shape.




R-OSSE [x,y] [mm]
x(t) [mm]
y(t) [mm]
r=0.1
r=0.3
r=0.5
r=0.7


R-OSSE [x,y] [mm]
x(t) [mm]
y(t) [mm]
m=0.7
m=0.8
m=0.9
m=1.0


R-OSSE [x,y] [mm]
x(t) [mm]
y(t) [mm]
b=0.0
b=0.2
b=0.4
b=0.6

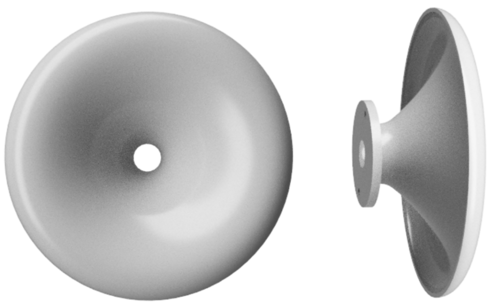

R-OSSE [x,y] [mm]
x(t) [mm]
y(t) [mm]
q=2
q=4
q=8

Practical design example

R-OSSE parameters

A sample waveguide[2] with 25.4 mm (1") throat is presented, given by the parameters listed below. This set of values results in a device that is 260 mm (~10") wide and slightly less than 80 mm (~3") long - see Fig. 4.

```
R = 130 mm          k = 1.8
r0 = 12.7 mm        r = 0.3
a0 = 7.5 deg        b = 0.3
a = 39 deg          m = 0.8
                    q = 3.7
```



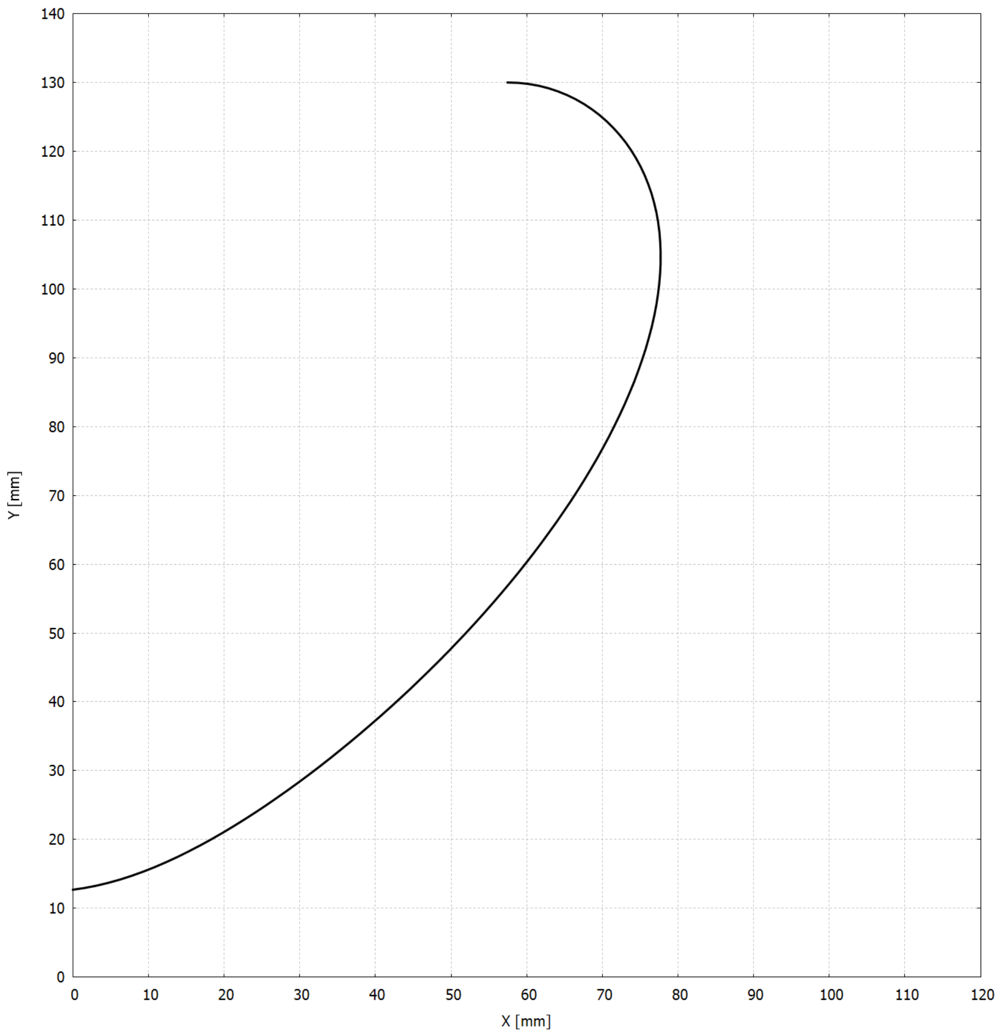


Fig. 4: Sample waveguide profile

2 This is an approximation of a device that has been around for some time, known as "ST260", made freely available for audio hobbyists, at the time constructed using a different and more complicated approach.

BEM simulation

The above axisymmetric device was numerically simulated in free field - as free standing with a 5 mm thick wall - via a boundary element metohd (BEM), using software ABEC by R&D Team[3] (Mr. Joerg Panzer). 100 frequency points between 200 Hz and 20 kHz were used for the calculation. Pulsating spherical cap corresponding to the given throat opening angle (a_0) was used as a source. The results are presented in the following graphs.

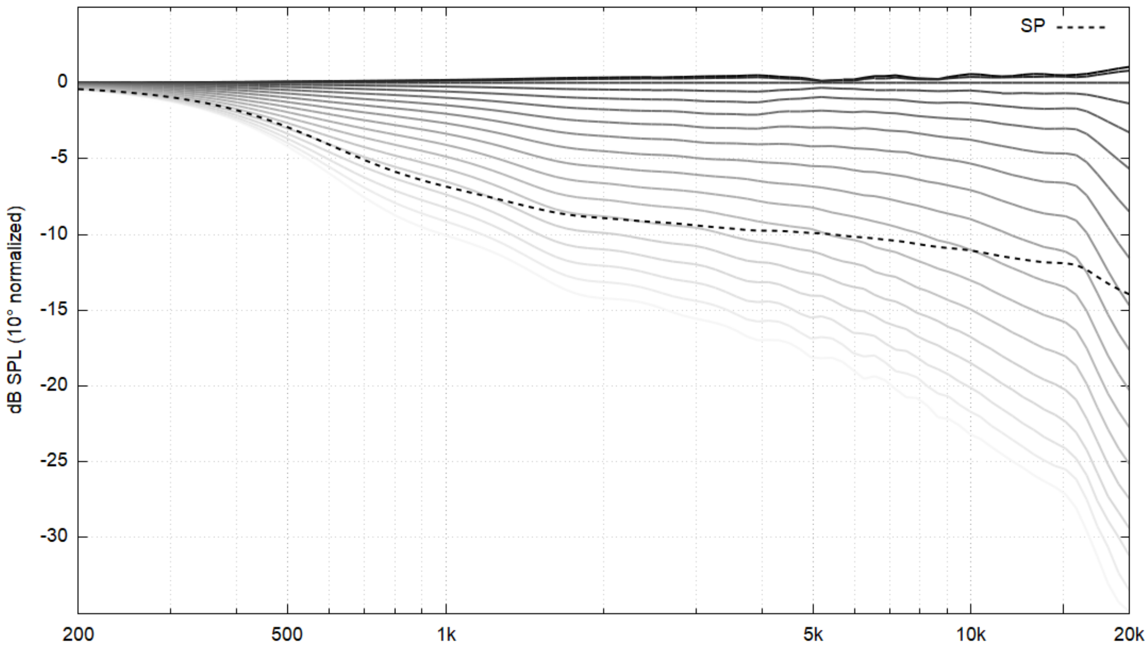


Fig. 5: BEM results - SPL polars, 0-90° / 5° step, 10° normalized

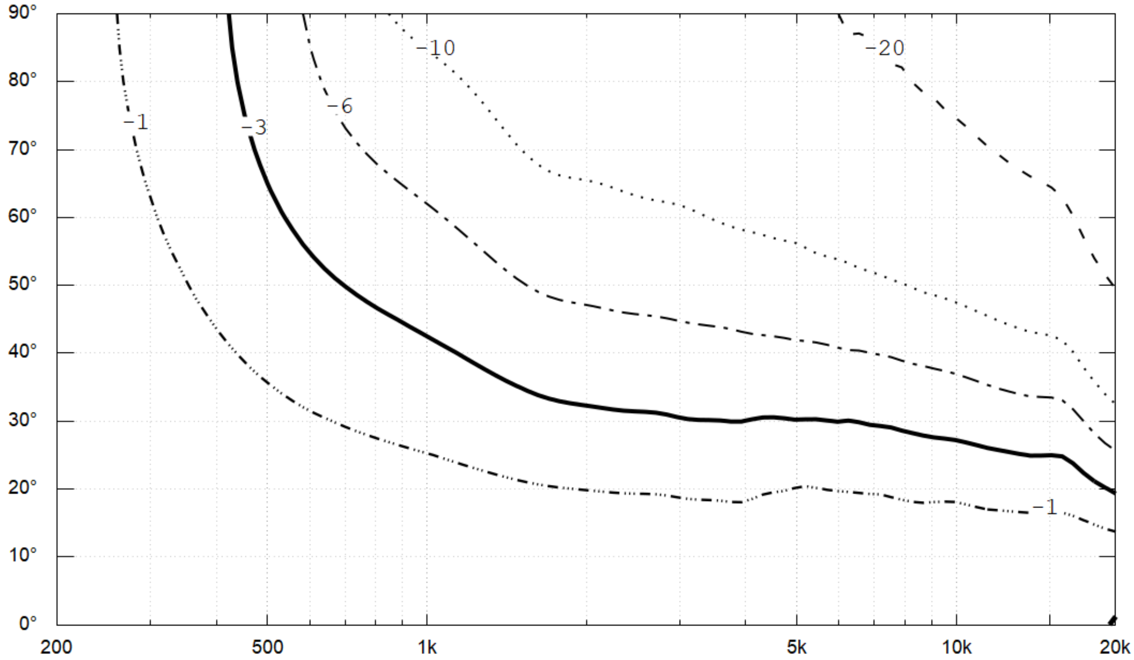


Fig. 6: BEM results - Polar map [dB SPL]

3 http://www.randteam.de

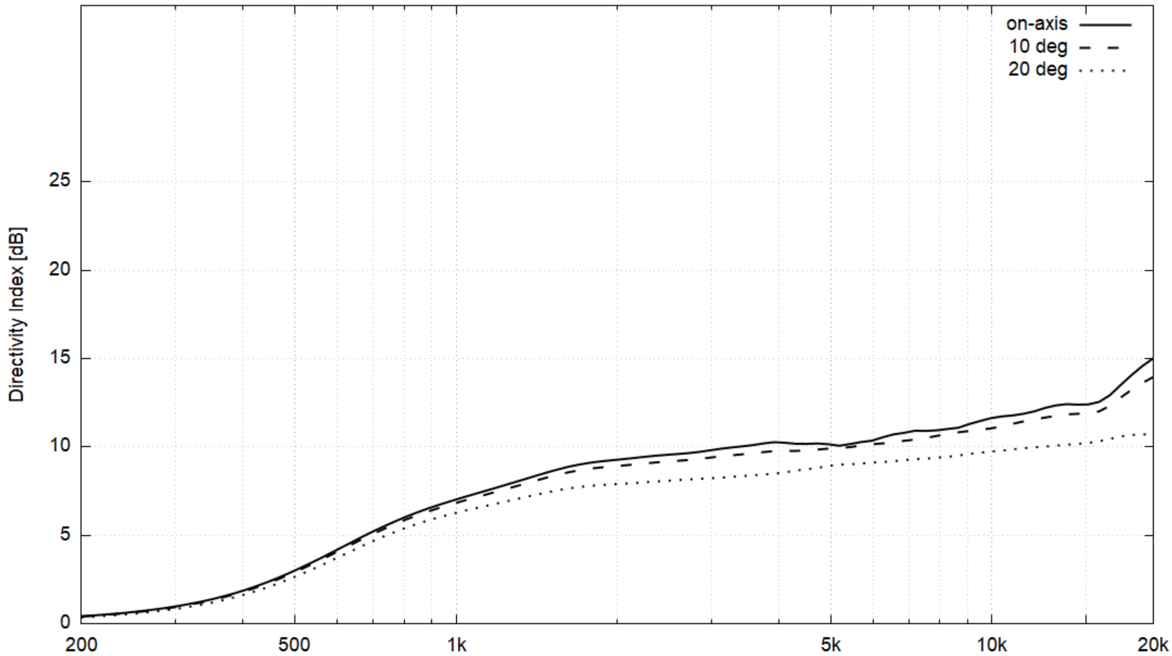


Fig. 7: BEM results - Directivity Index [dB]

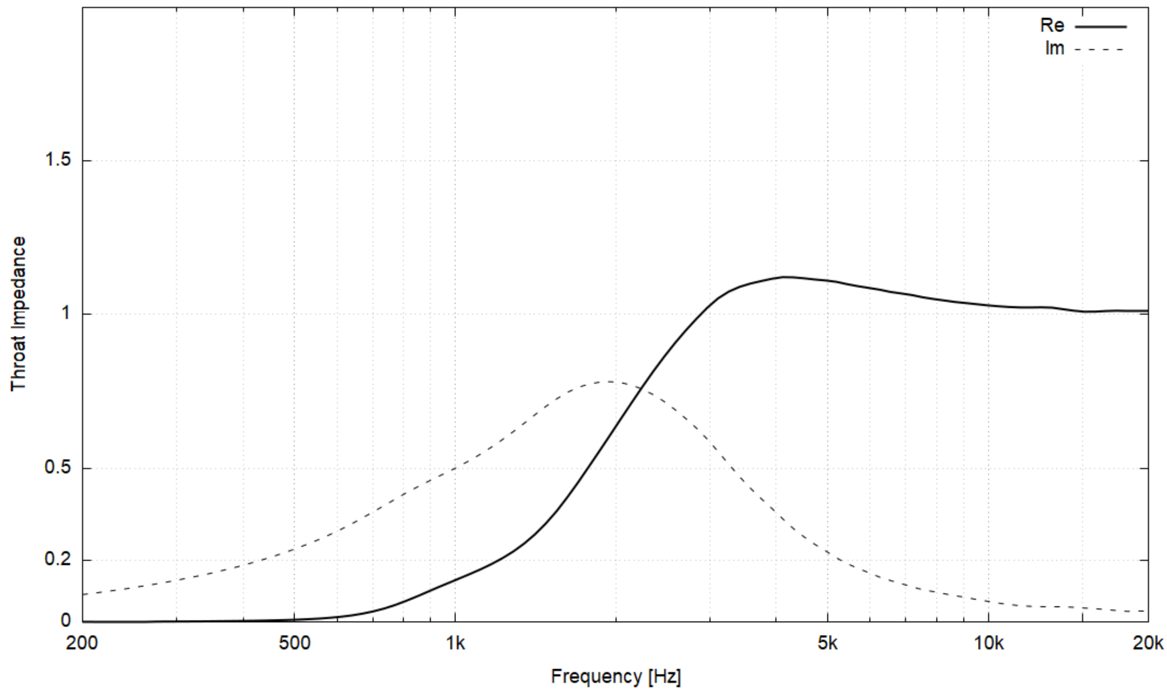


Fig. 8: BEM results - Throat acoustic impedance

Further Analysis

Throat opening angle

One of the parameters that can be adjusted easily is the throat opening angle a_0. In the above example the value is 7.5°, giving a total opening angle of 15°. Let's examine the results for $a_0 = 0°$ and $a_0 = 15°$ (Fig. 9). Note that the overall shape is changed by this adjustment.

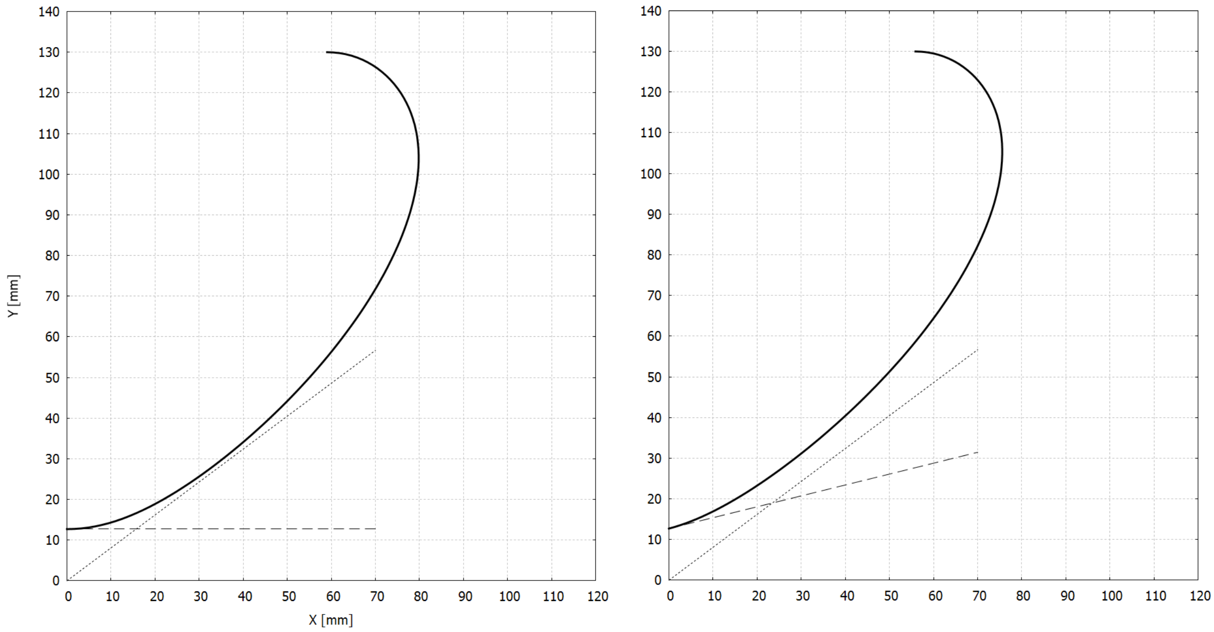


Fig. 9: Angle parameters: ***a_0*** *(dashed) and* ***a*** *(dotted)*

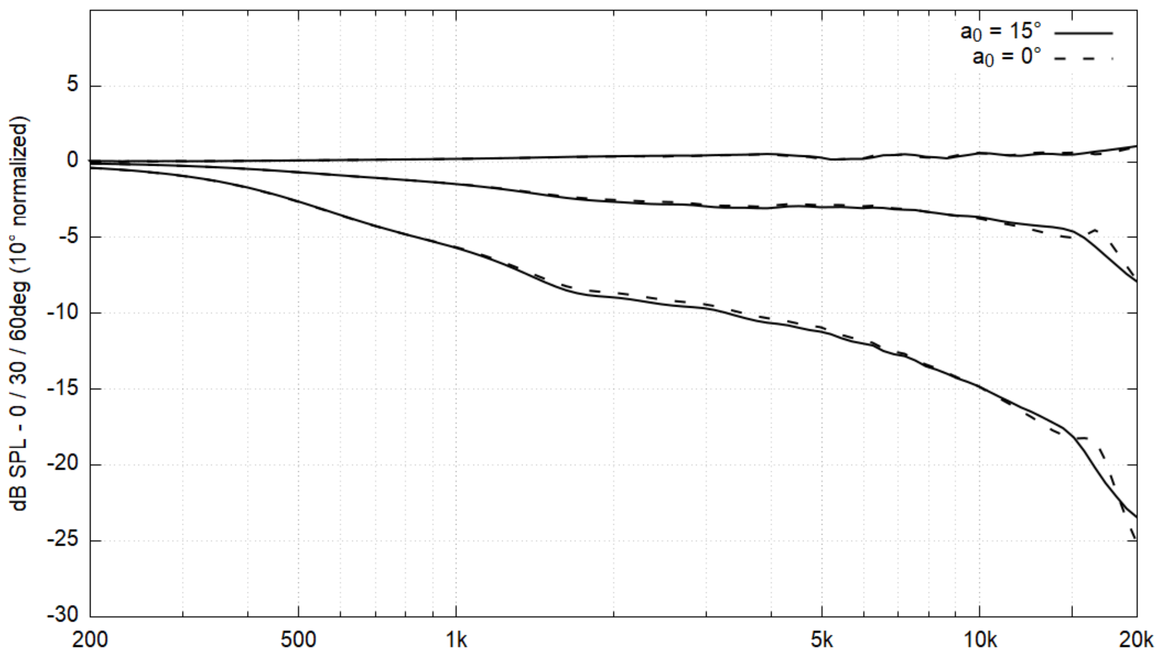


Fig. 10: Throat angle - SPL at 0 / 30 / 60°

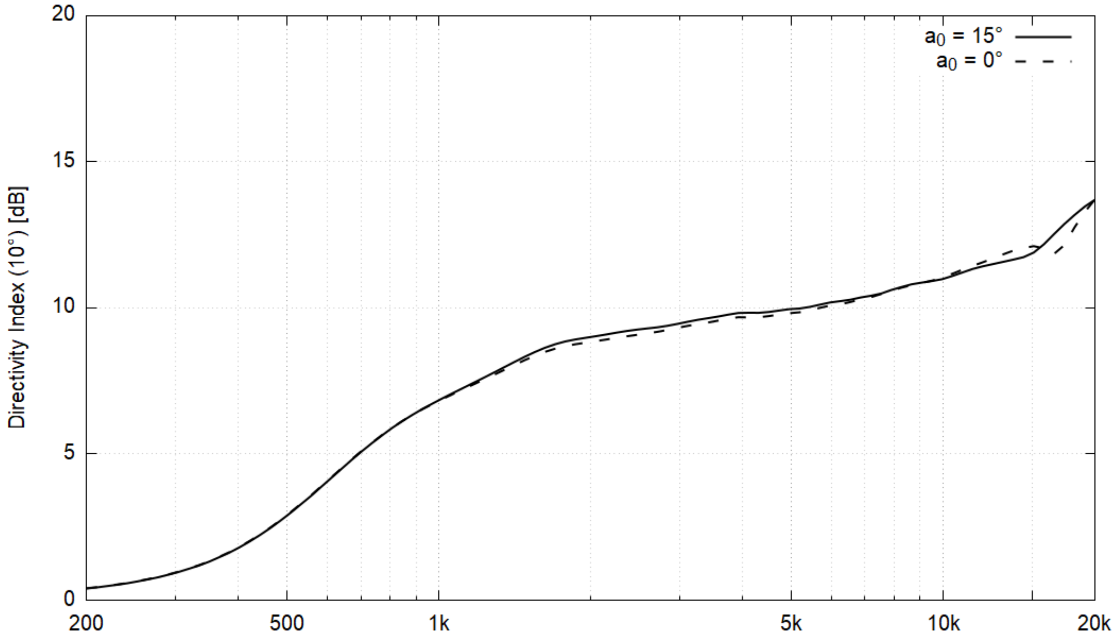


Fig. 11: Throat angle - Directivity Index at 10°

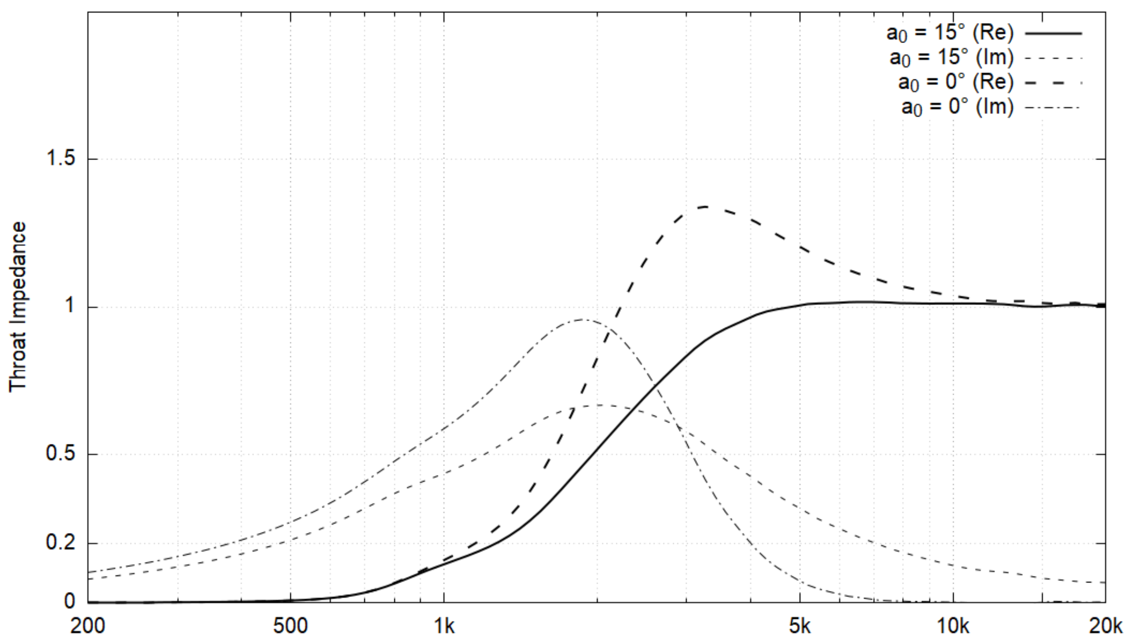
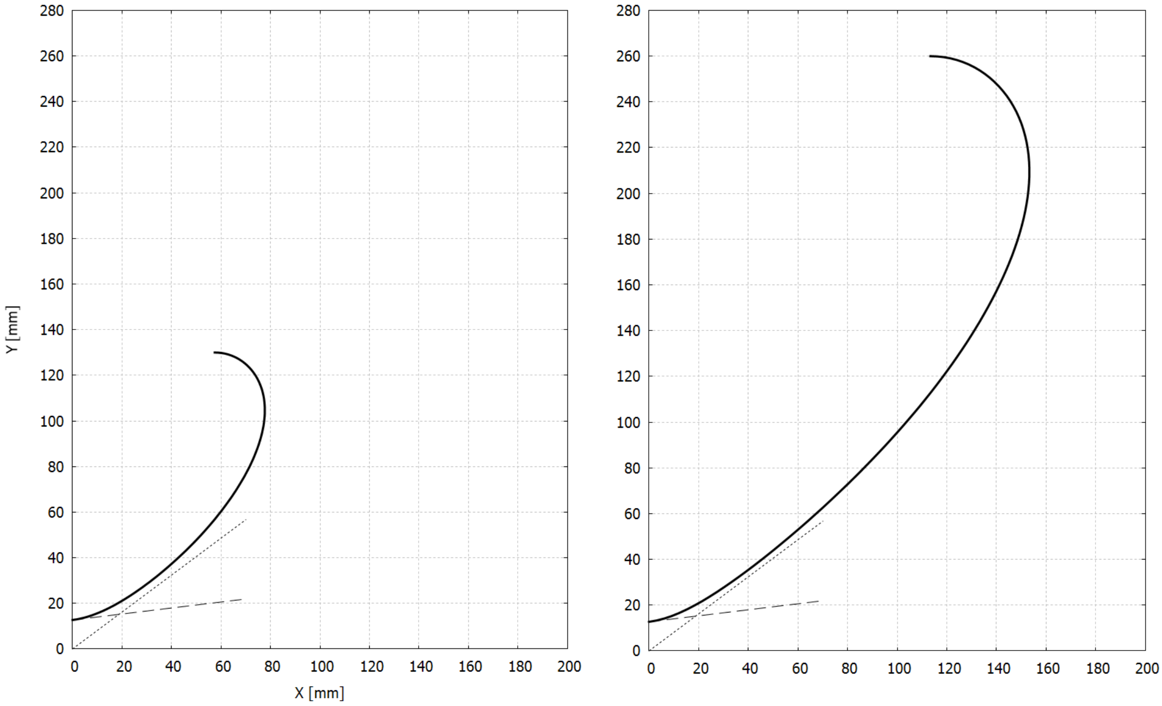


Fig. 12: Throat angle - Throat impedance

Waveguide size

The overall size of a waveguide is set directly with the parameter R, which is the outer radius. The above example is 260 mm in diameter, i.e. still pretty small in fact. The following graphs show results for a waveguide twice as large, i.e. 520 mm in diameter. (Only the parameter R has been changed, without any further optimization of the other parameters.)



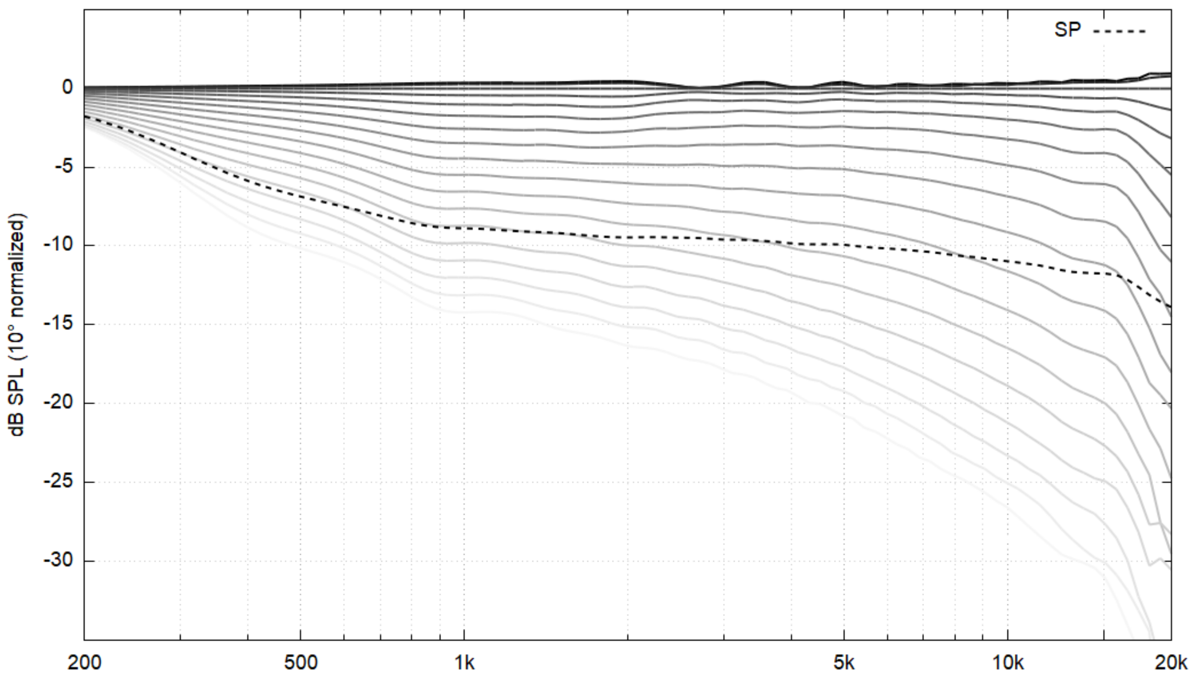


Fig. 13: SPL polars, 0-90° / 5° step, 10° normalized (R=260 mm)

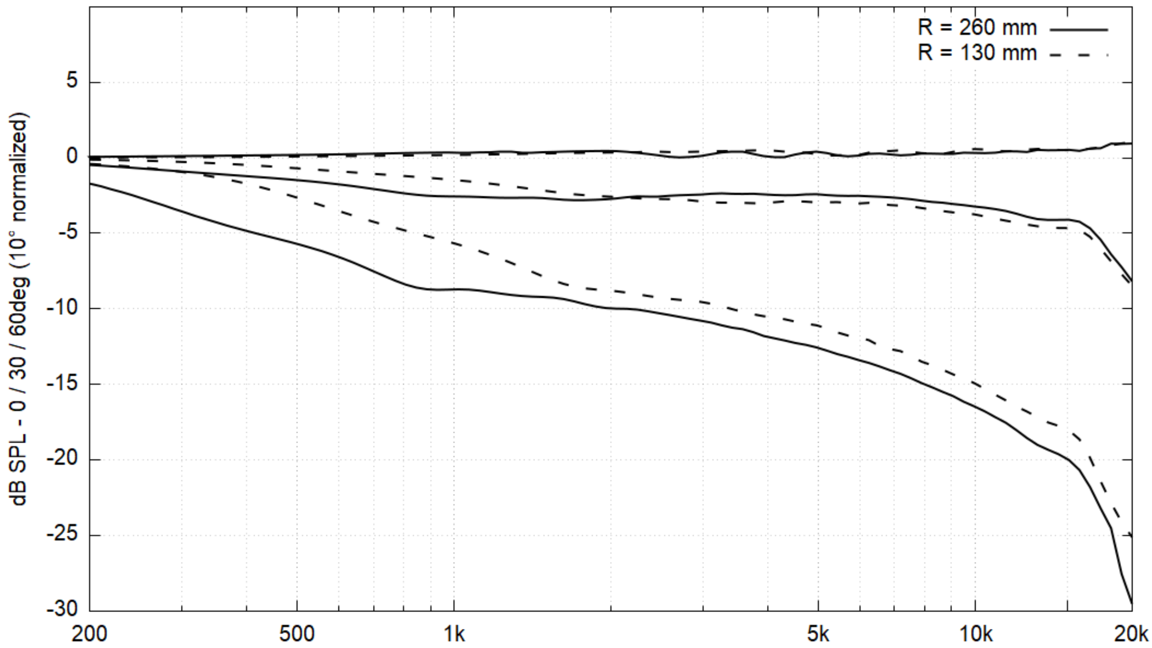


Fig. 14: Waveguide size - SPL at 0 / 30 / 60°

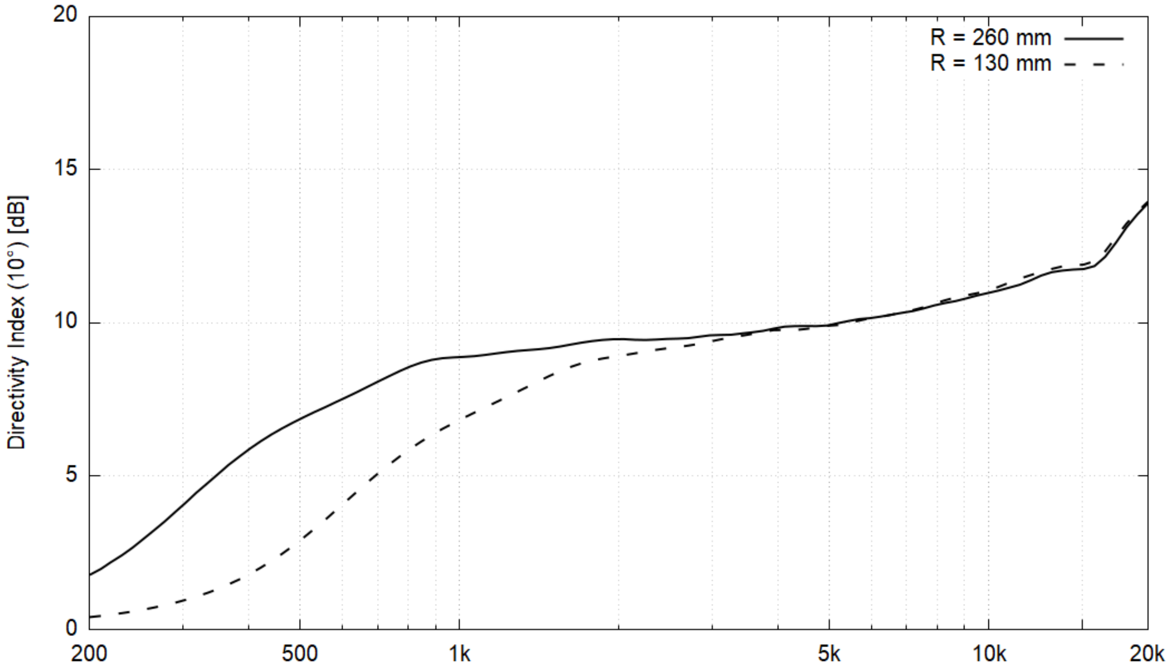


Fig. 15: Waveguide size - Directivity Index at 10°

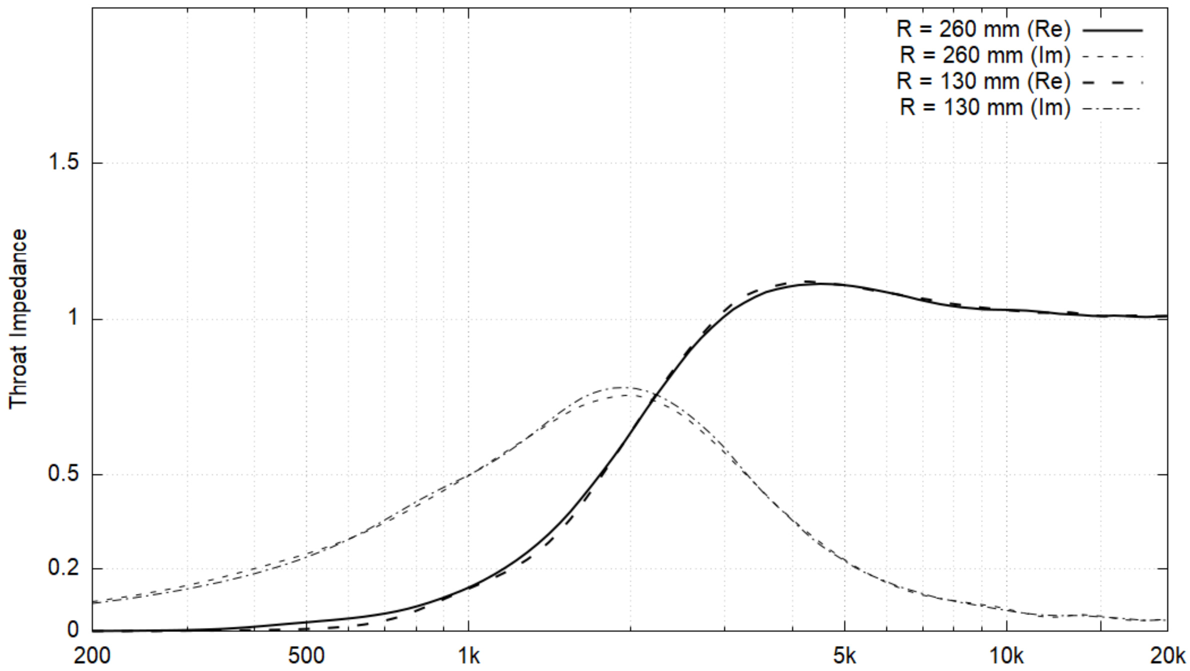


Fig. 16: Waveguide size - Throat impedance

Throat expansion factor

One of the parameters that greatly affects the performace is the "throat expansion factor", here denoted k. The whole idea behind this parameter is to use a throat curve that would correspond to a larger OS throat than the actual radius used. The value k = 1 gives the exact OS throat, a value k > 1 gives a throat curve corresponding to k-times larger radius.

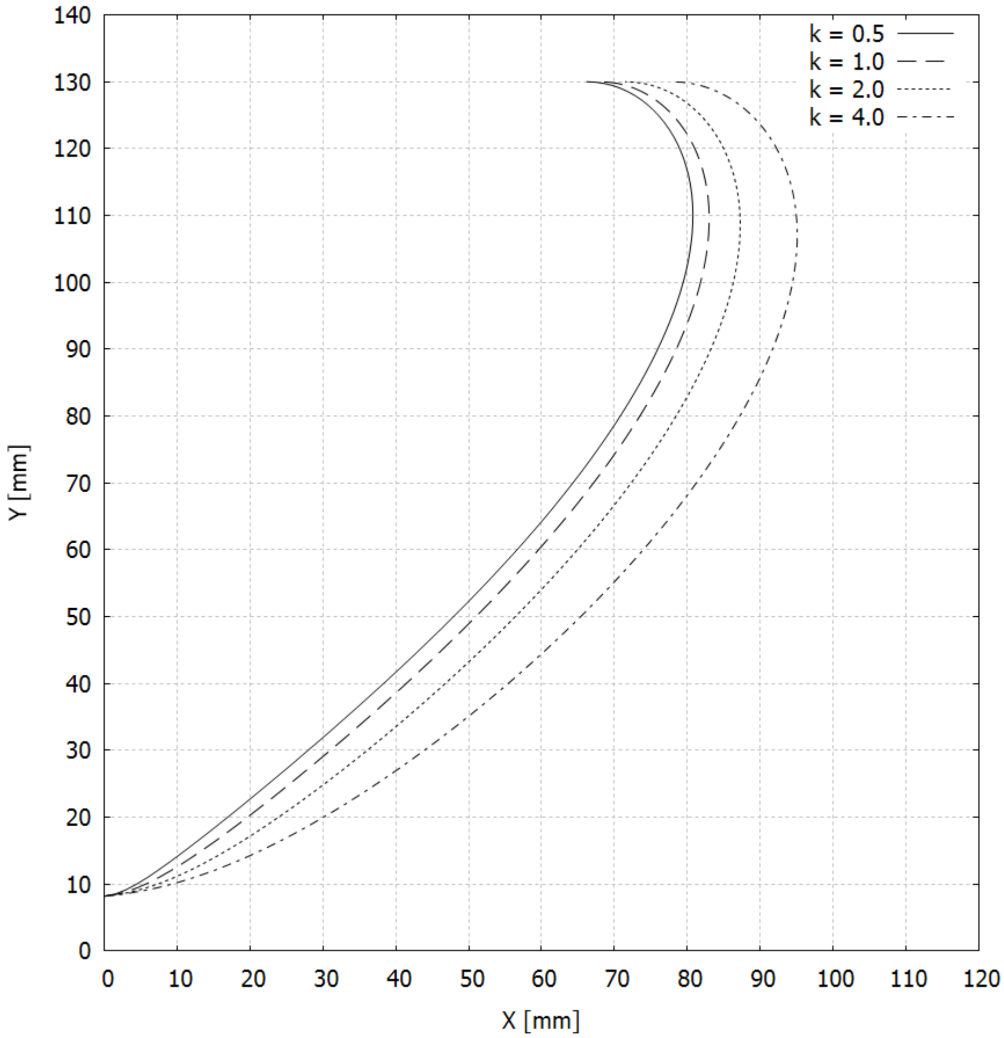


Fig. 17: Throat expansion factor

In the following figures BEM results are shown for k = 0.5, 1, 2 and 4. By adjusting this value a desired DI slope towards high frequencies can be set, trading a gently rising DI for an overall smoothness and a mildly higher throat impedance.

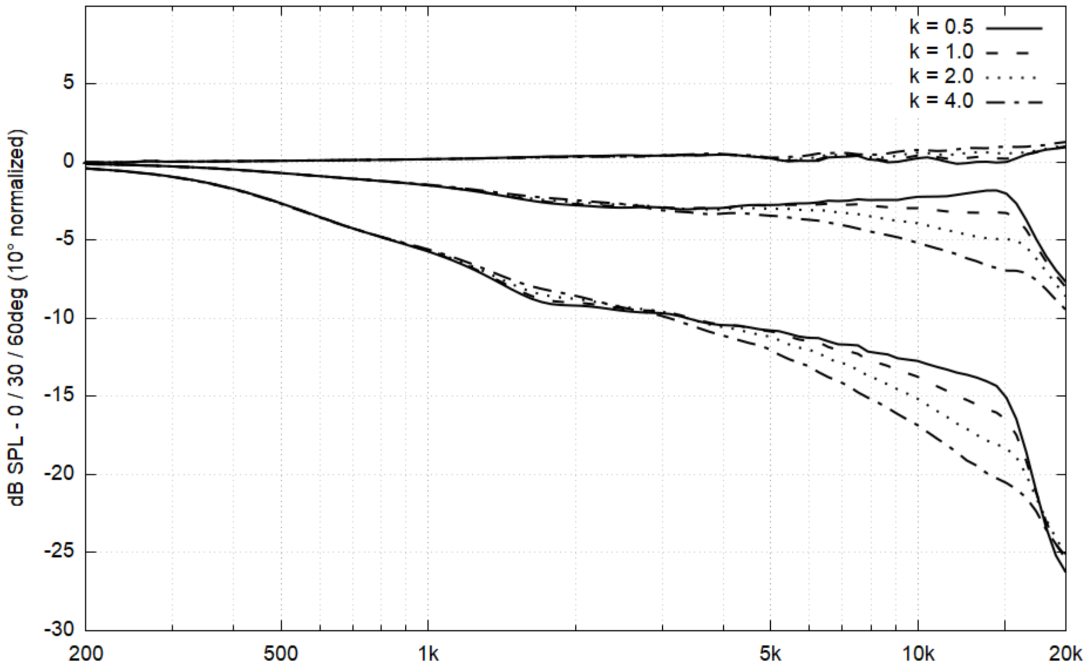


Fig. 18: Throat expansion factor - SPL at 0 / 30 / 60°

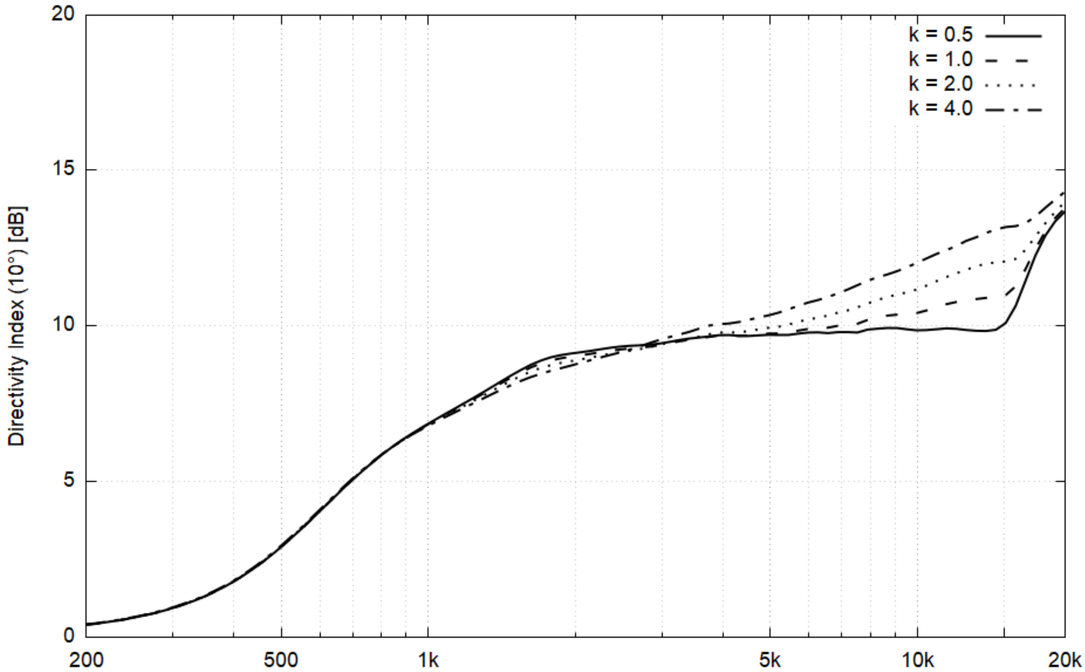


Fig. 19: Throat expansion factor - Directivity Index at 10°

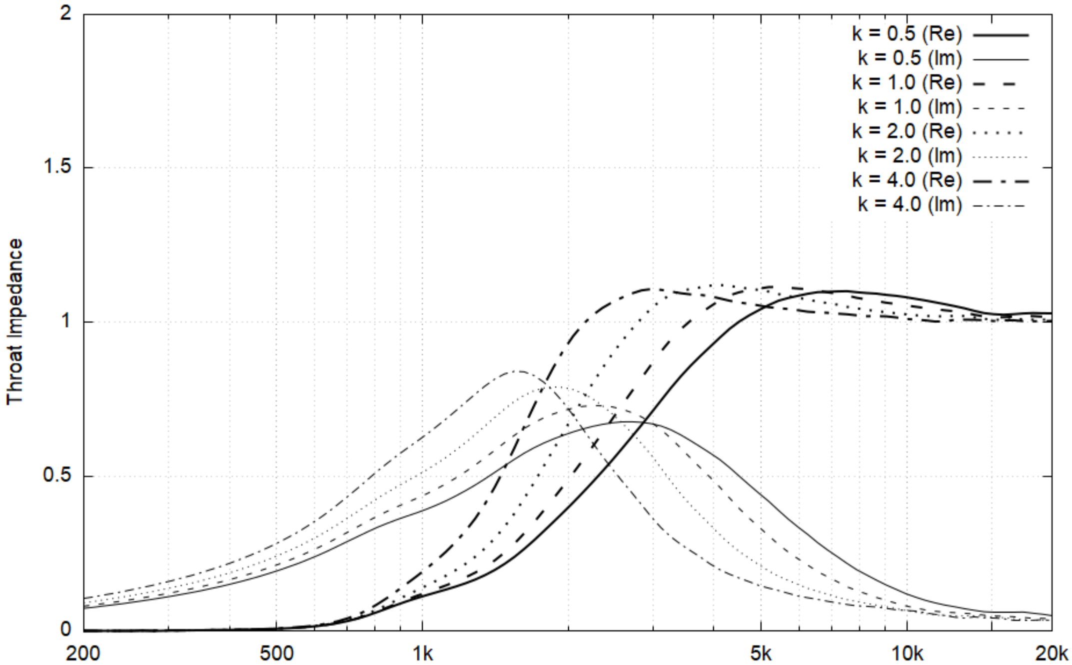


Fig. 20: Throat expansion factor - Throat Impedance

Throat diameter

In all the above examples a 1" throat was considered (r_0 = 12.7 mm). The following graphs show results for 1.4" (r_0 = 18 mm) and 2" (r_0 = 25 mm) throats, all other parameters unchanged. The larger waveguide from the previous excercise was used for this comparison (R = 260 mm).

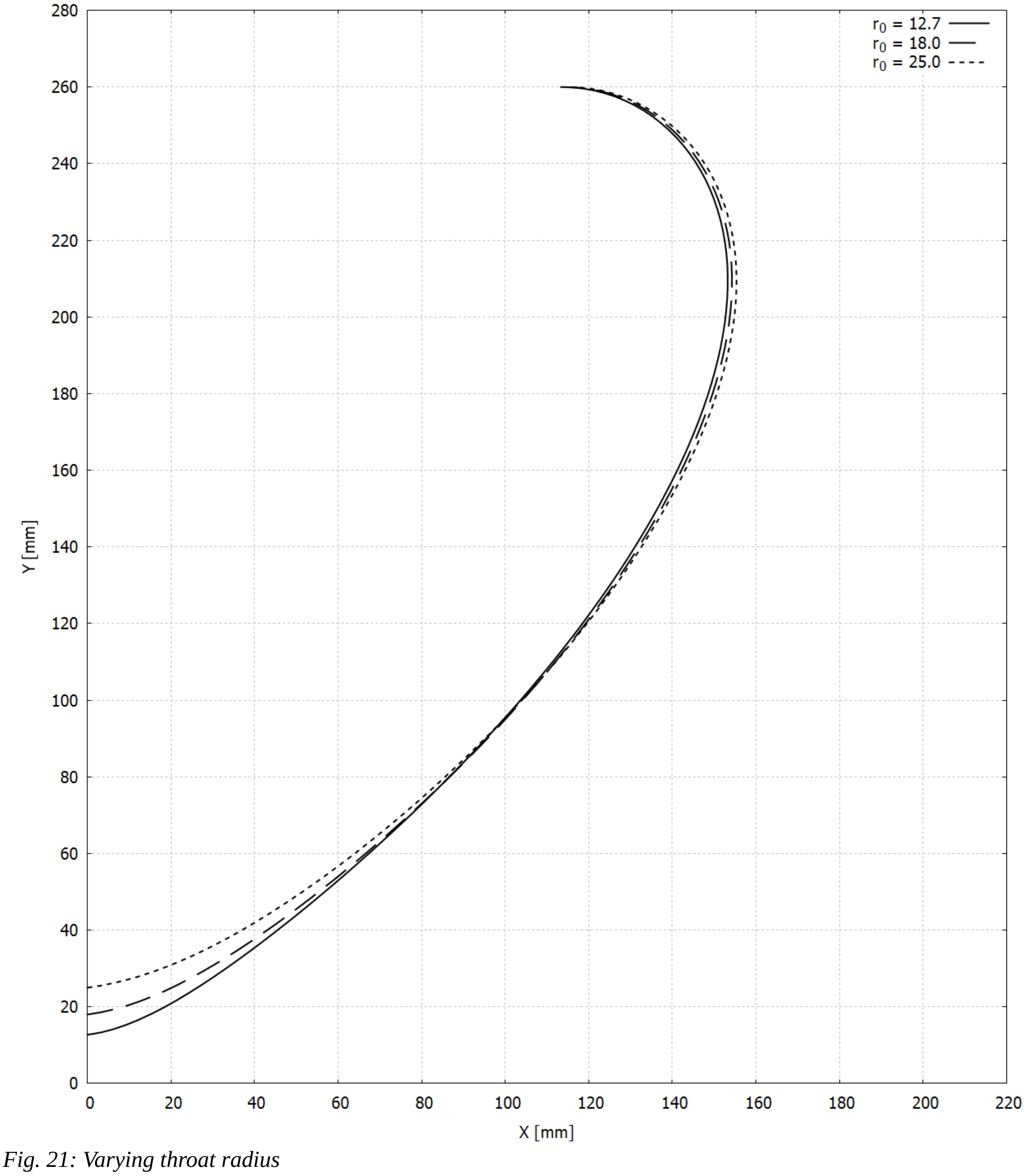


Fig. 21: Varying throat radius

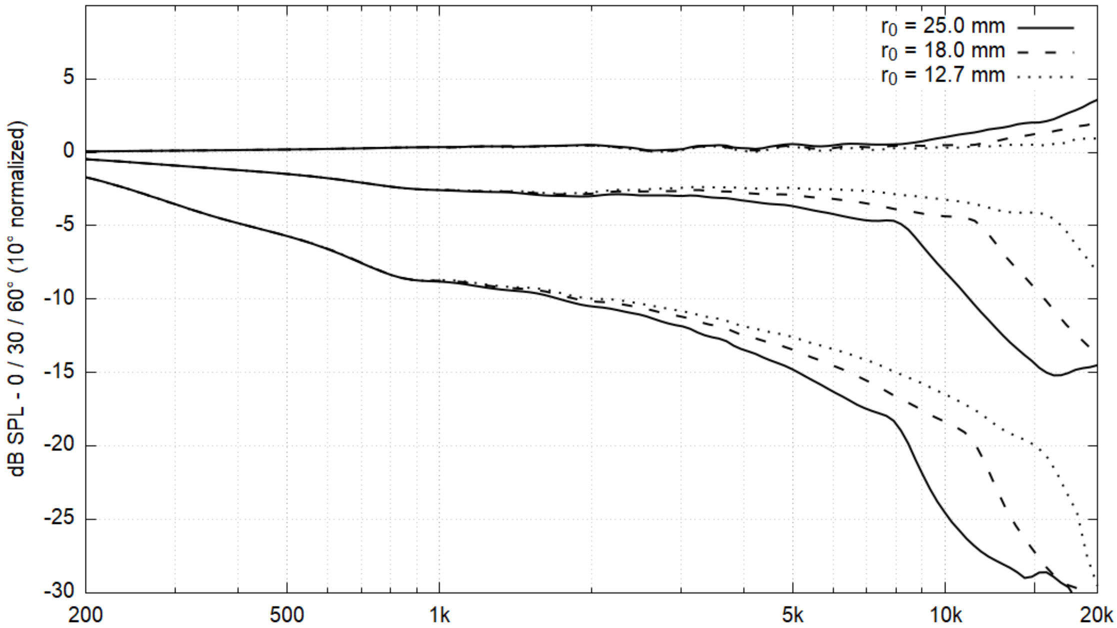


Fig. 22: Throat diameter - SPL at 0 / 30 / 60°

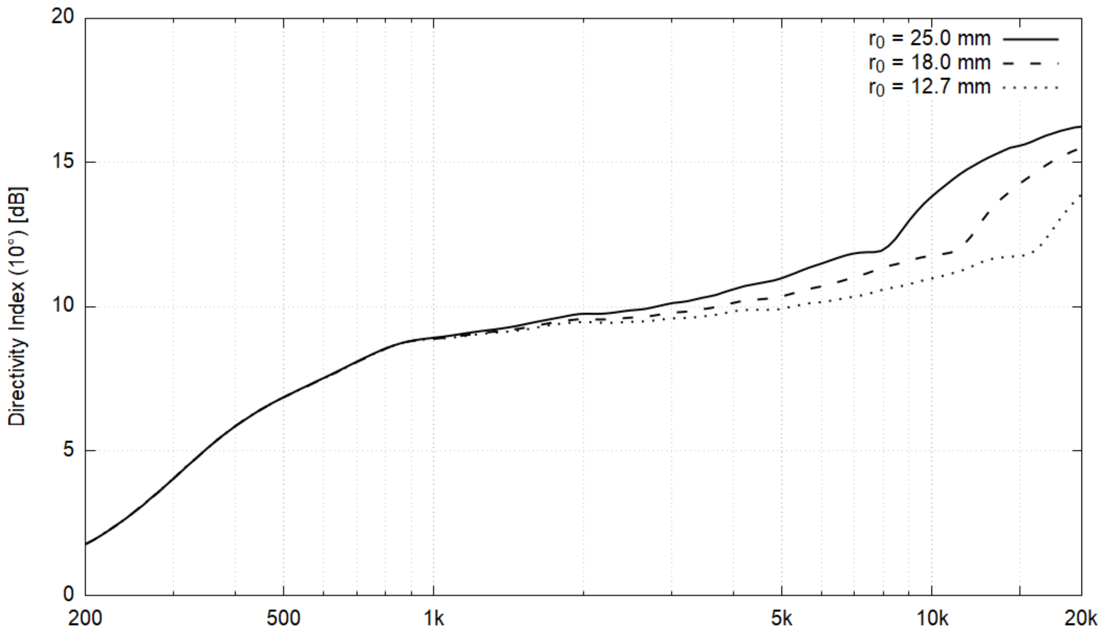


Fig. 23: Throat diameter - Directivity Index

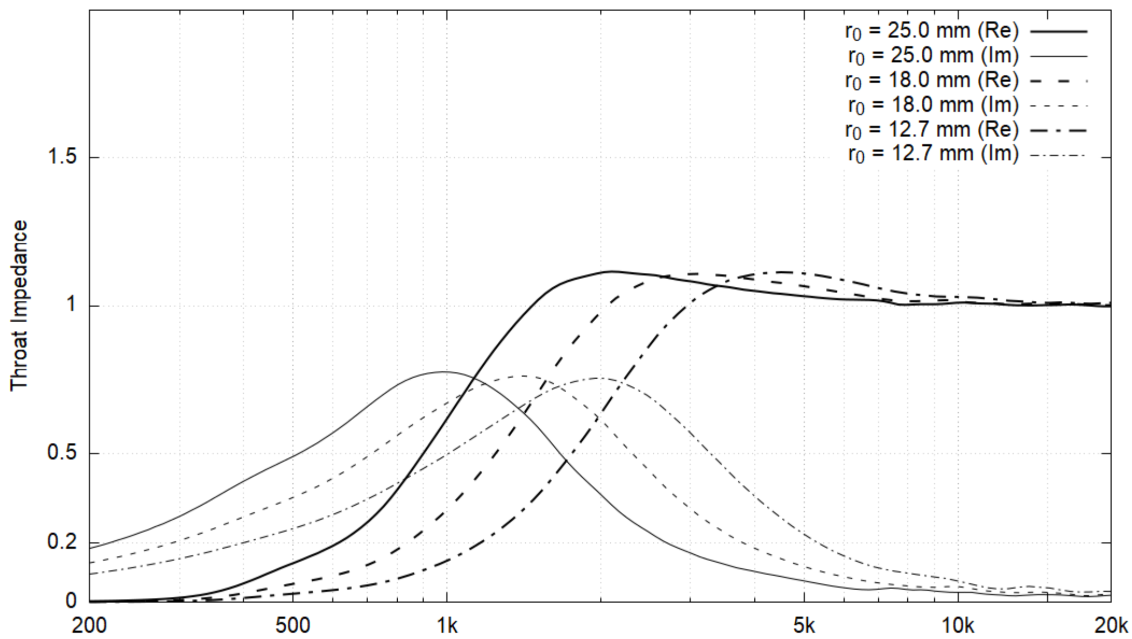


Fig. 24: Throat diameter - Throat impedance

Throat wavefront mismatch

In all the previous examples an ideal pulsating cap, corresponding to the throat opening angle, was used as a source in the simulations. However, many compression drivers contain a segment of a conical duct as their exit section and even if the slopes of the walls match at the driver/waveguide interface, there will be a wave reflection from this curvature discontinuity. In Fig. 25 there's such an interface - a hypothetical driver with a conical section 20 mm long is connected to the sample waveguide at x = 20 mm (r_0 = 12.7 mm, a_0 = 7.5°).

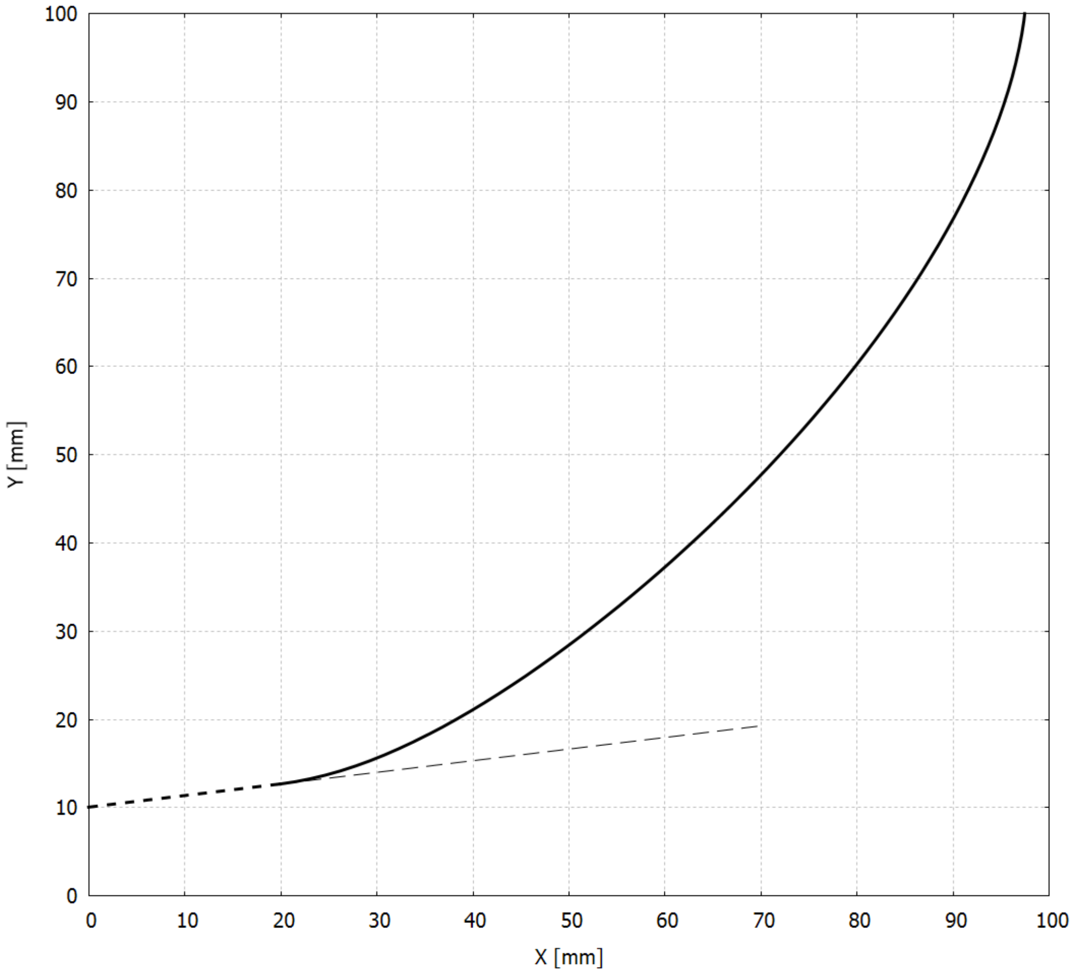


Fig. 25: Compression driver - waveguide interface (x=20 mm)

The resulting reflection is clearly indicated in the acoustic impedance plot, see Fig. 26.

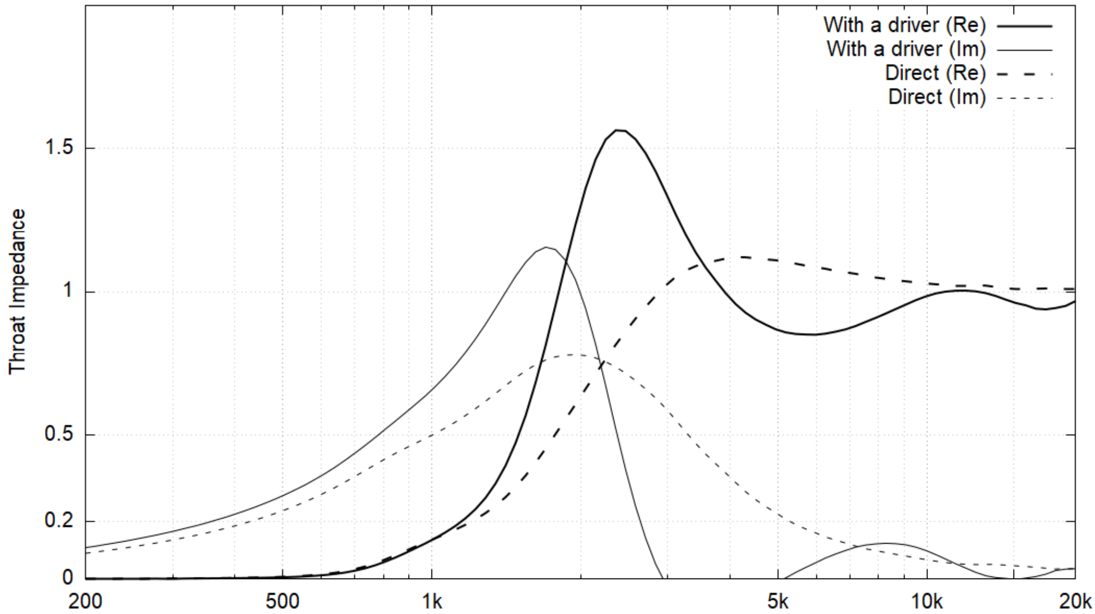


Fig. 26: Wavefront mismatch - Throat impedance

Throat impedances for some more lengths of the conical duct are presented in Fig. 27.

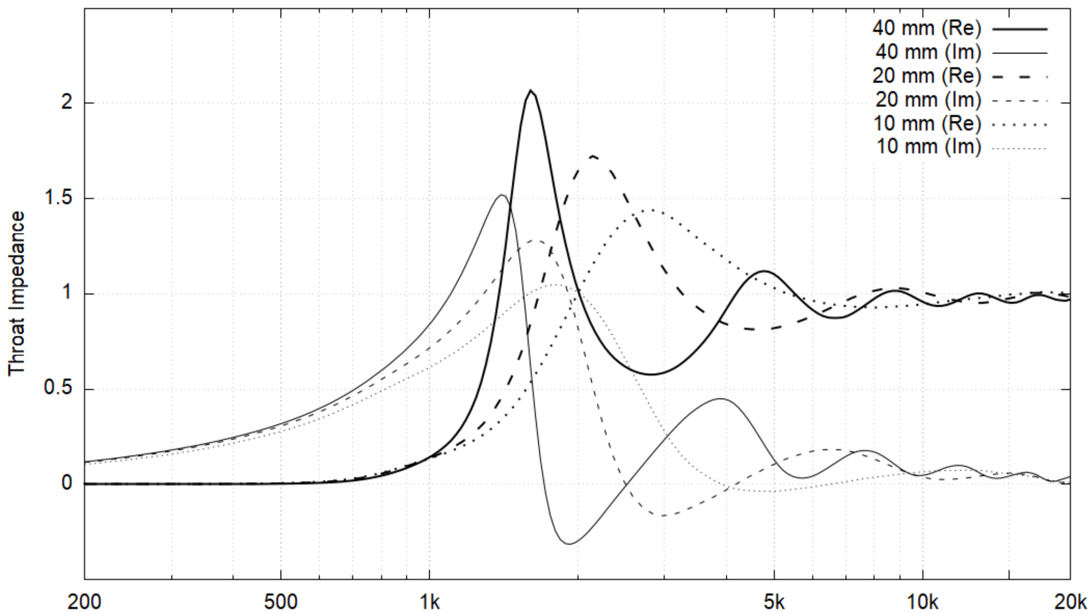


Fig. 27: Wavefront mismatch - Throat impedance (3 duct lengths)

There is virtually no impact on the normalized polar response (Fig. 28). However, such reflection will impact the absolute frequency response of the system by introducing additional ripples with the corresponding time domain effects.

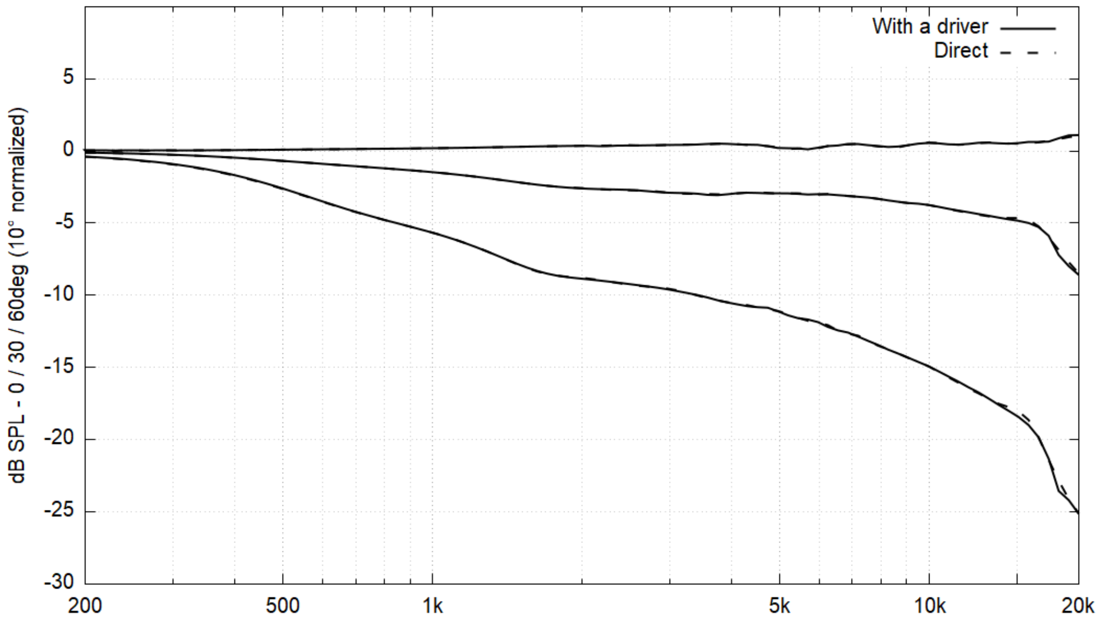


Fig. 28: Wavefront mismatch - normalized SPL at 0 / 30 / 60°

With a suitable compression driver, a possible improvement would be a ring insert, fitted into the conical duct of the driver, making one smooth waveguide contour, from a phase plug termination all the way into the free air (Fig. 29).

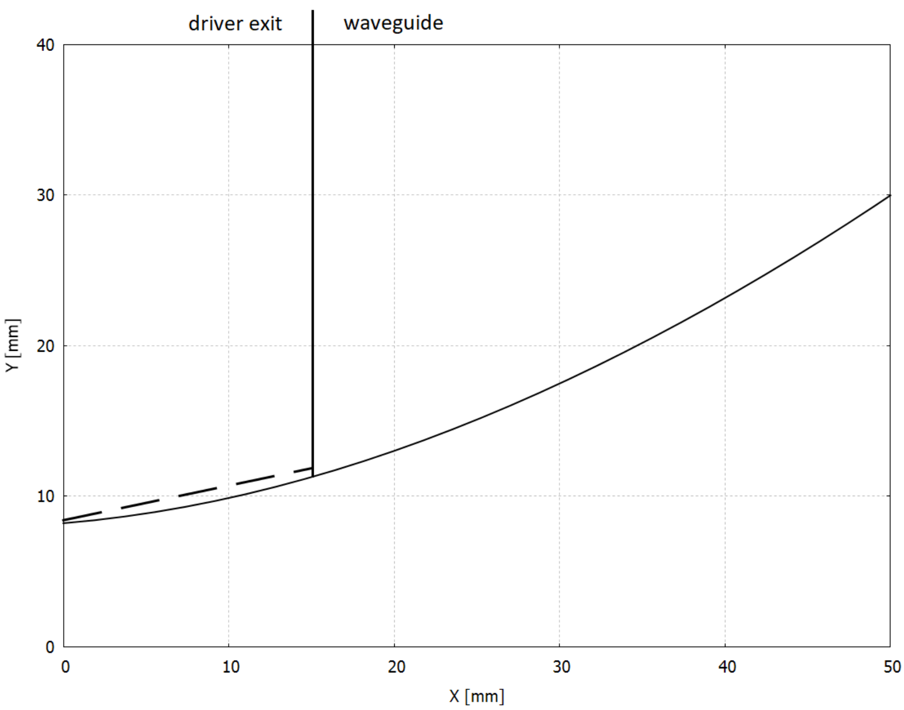


Fig. 29: Wavefront mismatch - Concept of a ring insert

To see a possible effect of the reflection caused by the wavefront mismatch, the sample waveguide was simulated including a (simplified) lumped element model of a 1" compression driver, i.e. including its mecahnical and electrical parts. The whole system was then simulated, driven with a voltage source.

Fig. 30 shows a simulation as if the driver had no conical exit section, i.e. driving the waveguide directly with its phase plug exit (supposedly producing an ideal spherical wave for $a_0 = 7.5°$).

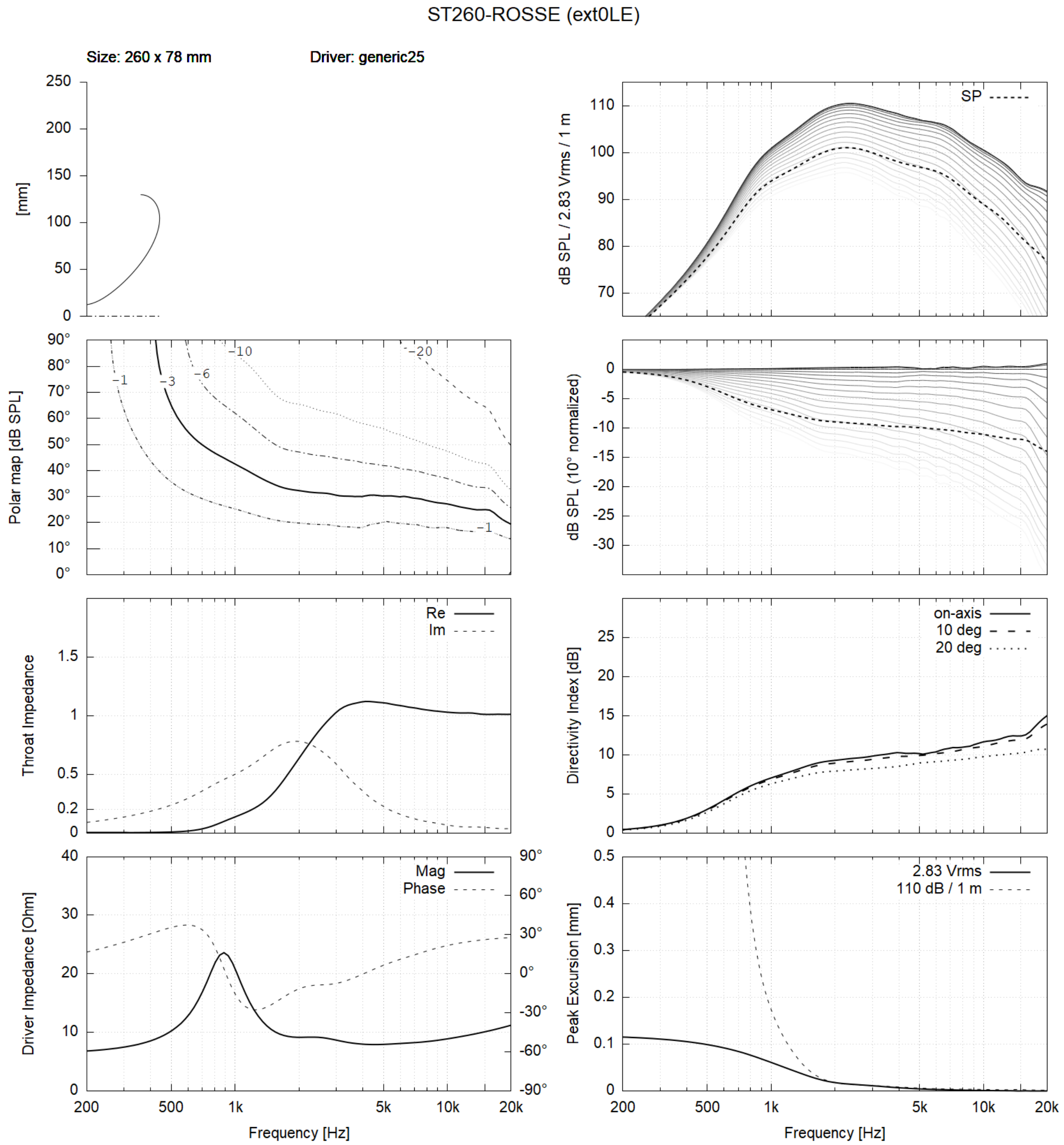


Fig. 30: CD direct drive / no duct

Fig. 31 shows the driver with a 20 mm conical duct. Although not very severe, the added resonances are clearly noticeable in the frequency response.

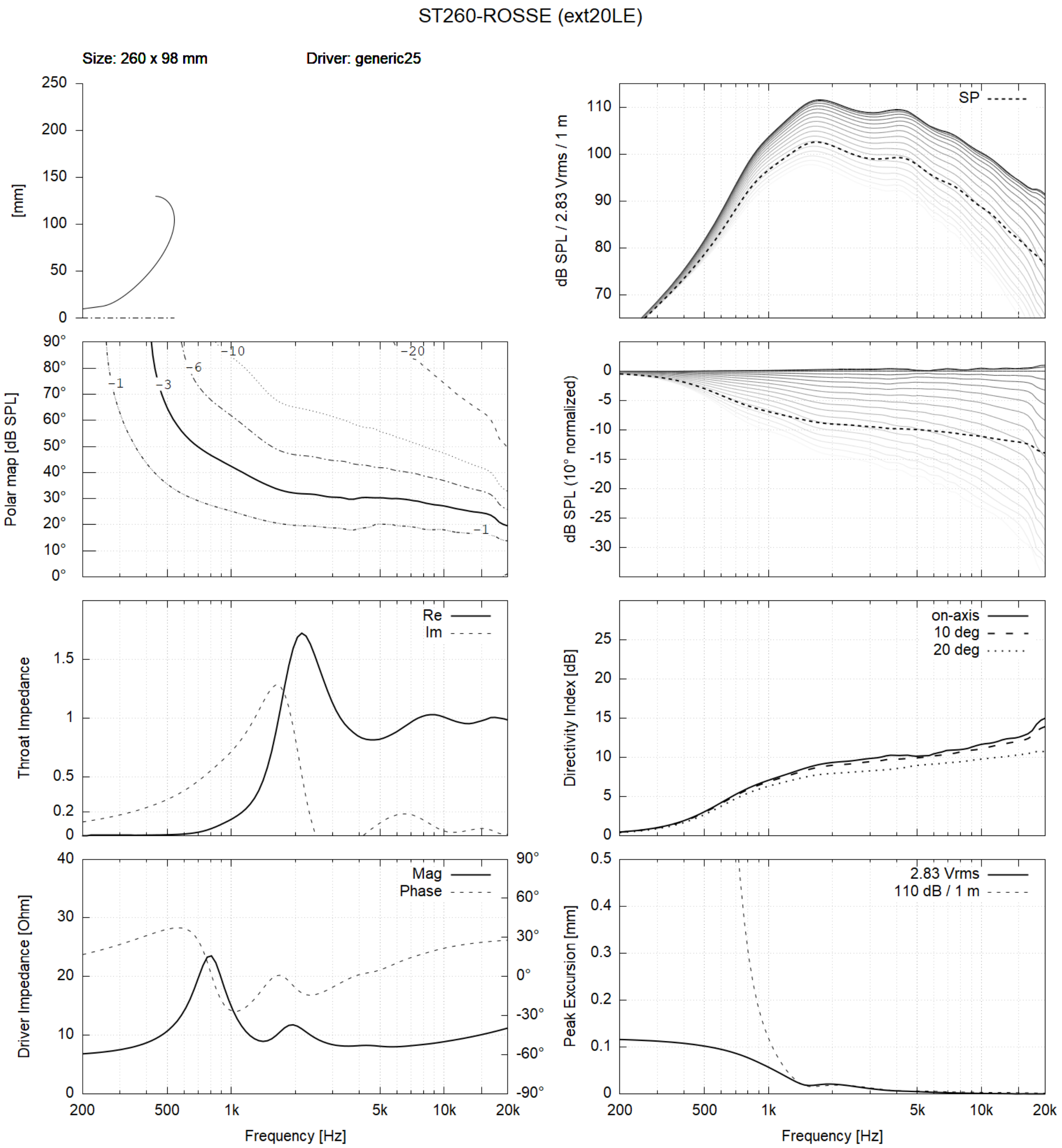


Fig. 31: CD + conical duct 20 mm

Fig. 32 shows the driver with a 40 mm conical duct. Although the higher acoustic impedance increases the output of the system on the lower end of its passband by several dB, it brings additional resonances that must be dealt with somewhere in the signal path.

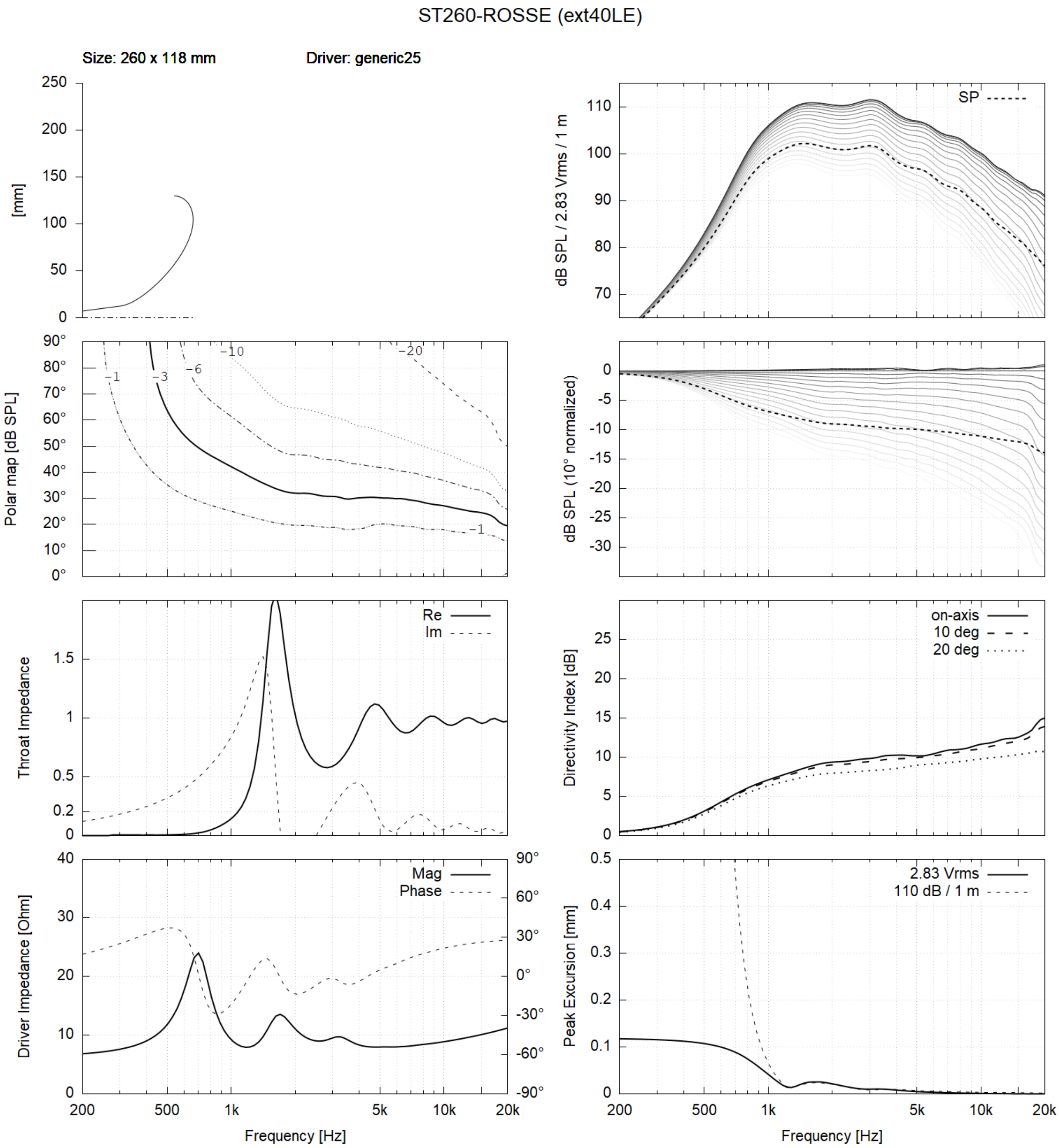


Fig. 32: CD + conical duct 40 mm

Finally, Fig. 33 displays a ring insert in action, which is effectively a waveguide starting with a smaller throat diameter. It does several things - eliminates the reflection, smoothes the overall acoustic response and also increases the system output by a bit.

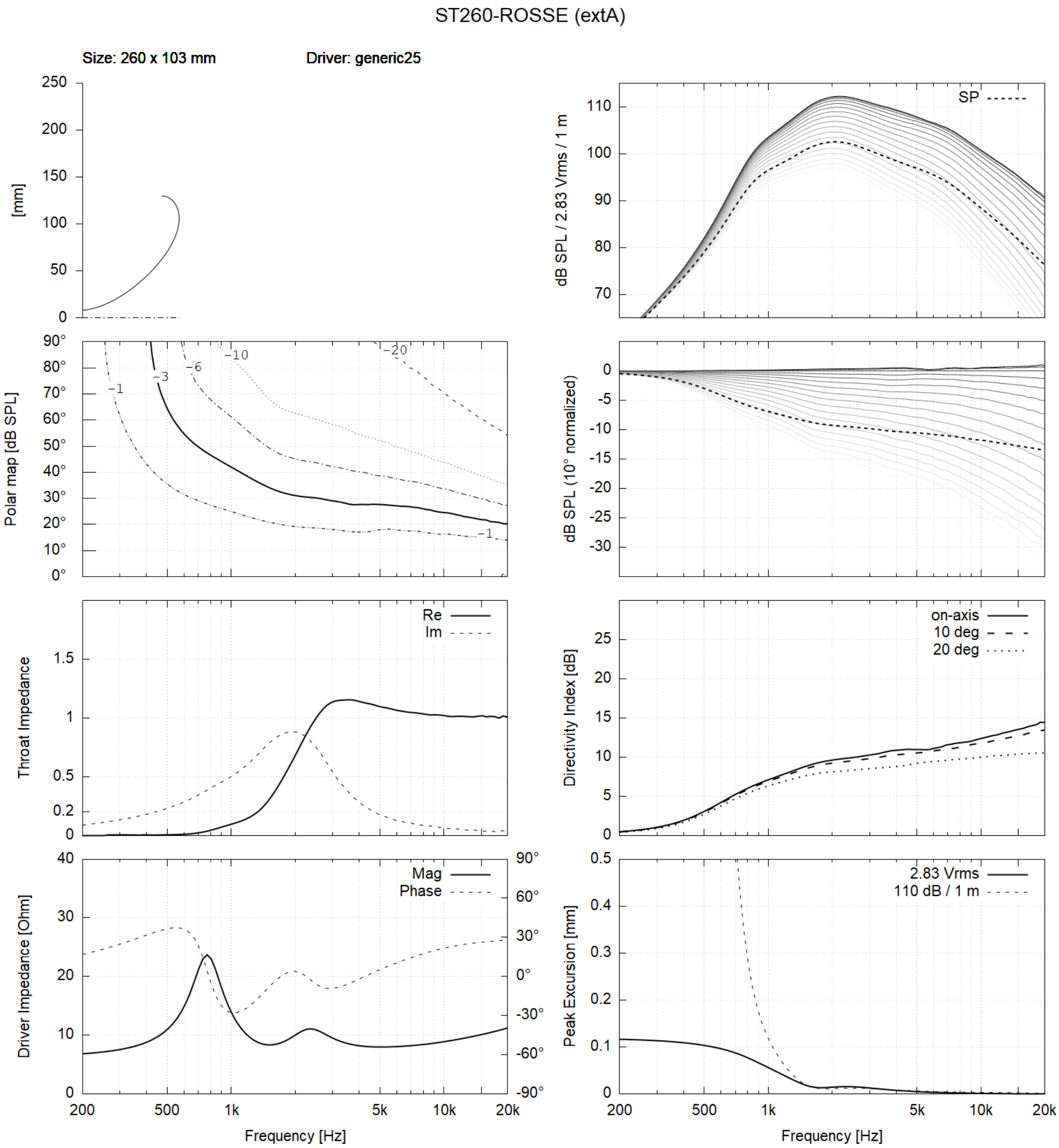


Fig. 33: Proposed ring insert

Ath script code

For a reference, this is the Ath[4] script code that was used to create the above waveguide BEM mesh.

```
R-OSSE = {
  R = 130
  r0 = 12.7
  a0 = 7.5
  a = 39
  k = 1.8
  r = 0.3
  b = 0.3
  m = 0.8
  q = 3.7
}

Mesh.LengthSegments = 60
Mesh.AngularSegments = 8
Mesh.SubdomainSlices =
Mesh.WallThickness = 5

Source.Shape = 1

ABEC.SimType = 2
ABEC.SimProfile = 0
ABEC.MeshFrequency = 43000
ABEC.NumFrequencies = 100
ABEC.Abscissa = 1
ABEC.f1 = 200
ABEC.f2 = 20000

ABEC.Polars:SPL = {
  MapAngleRange = 0,180,37
  NormAngle = 10
}

Output.ABECProject = 1
Output.STL = 0

Report = {
  Title = ST260-ROSSE
  Width = 1600
  Height = 1000
}
```

4 Ath design tool - https://at-horns.eu